Q3 EUF 2014 2sem

$$\rho(\nu)\,d\nu = \frac{8\pi\nu^2}{c^3} \cdot \frac{d\nu}{e^{h\nu/kT}-1}$$ densidade de energia

a) $\lambda = \frac{c}{\nu} \Rightarrow \nu = \frac{c}{\lambda}$ $\qquad B_\nu = \frac{2\pi\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_BT}}-1}$

$$d\lambda = -\frac{c\,d\nu}{\nu^2} \Rightarrow d\nu = -\frac{d\lambda\,\nu^2}{c} = -\frac{d\lambda\,c}{\lambda^2} \qquad E = B_\nu\,d\nu = \left(\frac{c}{\lambda}\right)^3 \cdot \left(-\frac{d\lambda\,c}{\lambda^2}\right)$$

$$\rho(\lambda)\,d\lambda = -\frac{8\pi}{\lambda^4} \frac{d\lambda}{\left(e^{\frac{hc}{\lambda k_BT}}-1\right)}$$

b) $\lambda \to \infty$, $T \to \infty$ $\gg 1$

$$\rho(\lambda)\,d\lambda = -\frac{8\pi\,d\lambda}{hc\lambda^4 + hc} \cdot \lambda k_BT$$

$$e^{\frac{hc}{\lambda k_BT}} \approx 1 + \frac{hc}{\lambda k_BT}$$

$$\boxed{\rho(\lambda)\,d\lambda = -\frac{8\pi\,d\lambda\,k_BT}{4\lambda^3 \cdot hc}}$$

c) Stefan-Boltzmann law $\Rightarrow j^* = \sigma T^4$ densidade

$$R_T = \frac{P}{A} = \int R(\lambda)\,d\lambda \int d\Omega$$ ângulo sólido $\qquad R_T = \sigma T^4 \Rightarrow$ power radiated from a black body

$(P = 4R_T)$ $\qquad R(\lambda) = c \cdot \frac{\rho(\lambda)}{4}$

$$= \int_0^\infty -\frac{8\pi c}{\lambda^4} \frac{d\lambda}{\left(e^{\frac{hc}{\lambda k_BT}}-1\right)} = /$$

Q4.

A: $v_0 \rightarrow$, $-v_0 \leftarrow$ Inertial

B: Rest, $-v'$, $\leftarrow$

energia cinética

$K = E - mc^2$

energia total, energia repouso

$\hookrightarrow \gamma m c^2$

a) $v_x' = \dfrac{v_x - V}{1 - \dfrac{V v_x}{c^2}} = \dfrac{-v - v}{1 + \dfrac{v^2}{c^2}} = \boxed{\dfrac{-2v}{1 + \dfrac{v^2}{c^2}}}$ ✓

eles param depois da colisão

b) B Antes

$p_i = \gamma m \cdot v_x' = -\dfrac{m 2v}{1 + \dfrac{v^2}{c^2}} \cdot \dfrac{1}{\sqrt{1 - \dfrac{v_x'^2}{c^2}}} = \dfrac{-m v_x'}{\sqrt{1 - \dfrac{v_x'^2}{c^2}}}$

Após,

$v_B = v$

$v_A = 0$

$p_f = \gamma_2 M v = \dfrac{2 M v}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

c) conservação momento $\Rightarrow$ $\cancel{\dfrac{-m 2v}{1 + \dfrac{v^2}{c^2}}} = Mv \Leftrightarrow M = \cancel{\dfrac{-m2}{1 - \dfrac{v^2}{c^2}}}$

visto z pq 2 partículas com velocidade iguais

A

$E_i = \gamma m_0 c^2 = 2 \cdot \dfrac{m_0 c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ $\Rightarrow$ B: $E_1 = \gamma_1 m_0 c^2 + m_0 c^2 = \left(\dfrac{m_0}{\sqrt{1 - \dfrac{v_x'^2}{c^2}}} + m_0\right) c^2$

B

$E_f = 2\gamma_2 M c^2 = \dfrac{2Mc^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$E_i = E_f$

$\gamma_1 m_0 \cancel{c^2} + m_0 \cancel{c^2} = 2\gamma_2 M \cancel{c^2}$

$$\boxed{M = \frac{m_0}{2\gamma_2}(\gamma_1 + 1)}$$

Q9.

$V = \frac{1}{2} m\omega^2 (x^2+y^2+z^2)$

a)

$-\frac{\hbar}{2m}\nabla^2\psi + V\psi = E\psi$

$\psi' = -\alpha r C e^{-\frac{\alpha r^2}{2}}$

$\psi'' = -\alpha C e^{-\frac{\alpha r^2}{2}} + \alpha^2 r^2 C e^{-\frac{\alpha r^2}{2}}$

suppose $\psi = Ce^{-\alpha r^2/2}$

$-\frac{\hbar}{2m}\left[-\alpha C \cancel{e^{-\frac{\alpha r^2}{2}}} + \alpha^2 r^2 C \cancel{e^{-\frac{\alpha r^2}{2}}}\right] + \left(\frac{1}{2}m\omega^2 r^2 - E\right) C \cancel{e^{-\frac{\alpha r^2}{2}}} = 0$

$-\alpha \cancel{C} + \alpha^2 r^2 \cancel{C} - \frac{m^2\omega^2 r^2}{\hbar}\cancel{C} + \frac{2mE}{\hbar}\cancel{C} = 0$

1D

define:

$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right)$ and $\hat{a}^{+} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i\hat{p}}{m\omega}\right)$

$\hat{H}\psi = E\psi$; $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \Rightarrow \hat{H} = \hbar\omega\left[\frac{\hat{p}^2}{2m\hbar\omega} + \frac{m\omega\hat{x}^2}{2\hbar}\right]$

$\hat{a}\hat{a}^{+} = \frac{m\omega}{2\hbar}\left(\hat{x}^2 + \frac{\hat{p}^2}{(m\omega)^2} - \frac{i\hat{x}\hat{p}}{m\omega} + \frac{i\hat{p}\hat{x}}{m\omega}\right)$

$\hat{H} = \hbar\omega\left[\hat{a}^{+}\hat{a} + \frac{1}{2}\right]$

$= \frac{m\omega\hat{x}^2}{2\hbar} + \frac{\hat{p}^2}{2\hbar m\omega} + \frac{i}{2\hbar}[\hat{p},\hat{x}] = \frac{m\omega\hat{x}^2}{2\hbar} + \frac{\hat{p}^2}{2\hbar m\omega} + \frac{1}{2}$

$[\hat{x},\hat{p}] = i\hbar$

$$\hat{H} = \frac{\hbar\omega}{2}\left(2\hat{a}\hat{a}^{\dagger} + 1\right) \quad ; \quad [\hat{H}, a^{\dagger}] = \left(\hbar\omega\hat{a}\hat{a}^{\dagger} + \frac{\hbar\omega}{2}\right)a^{\dagger} - a^{\dagger}\cdot\left(\hbar\omega\hat{a}\hat{a}^{\dagger} + \frac{\hbar\omega}{2}\right)$$

$[\hat{a}, \hat{a}^{\dagger}] = +1$

$$= \hbar\omega\left(\hat{a}\hat{a}^{\dagger}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{a}^{\dagger}\right) + \cancel{\frac{\hbar\omega}{2}\hat{a}^{\dagger}} - \cancel{\hat{a}^{\dagger}\frac{\hbar\omega}{2}}$$

$$= \hbar\omega\left(\hat{a}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\right)\hat{a}^{\dagger} = \hbar\omega\,[\hat{a}, \hat{a}^{\dagger}]\,\hat{a}^{\dagger} = +\hbar\omega\hat{a}^{\dagger}$$

$$-\hat{a}^{\dagger}\hat{a} = -\frac{m\omega}{2\hbar}\left(\hat{x} - \frac{i\hat{p}}{m\omega}\right)\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right) = -\frac{m\omega}{2\hbar}\left[\hat{x}^2 + \frac{\hat{p}^2}{m\omega} - \frac{i\hat{p}\hat{x}}{m\omega} + \frac{i\hat{x}\hat{p}}{m\omega}\right] = +\frac{m\omega\hat{x}^2}{2\hbar} - \frac{\hat{p}^2}{2\hbar} - \frac{i}{2\hbar}\overset{+\frac{1}{2}}{[x, p]}$$

Ground state E_0

$$E_0 = \langle 0|\hat{H}|0\rangle = \langle 0|\frac{\hbar\omega}{2}\left(2\hat{a}^{\dagger}\hat{a} + 1\right)|0\rangle = \langle 0|\hbar\omega\hat{a}^{\dagger}\hat{a}|0\rangle + \langle 0|\frac{\hbar\omega}{2}|0\rangle$$

$$= \langle 0|\hbar\omega\hat{a}^{\dagger}(\hat{a}|0\rangle) + \frac{\hbar\omega}{2}\langle 0|0\rangle = 0 + \frac{\hbar\omega}{2} = \boxed{\frac{\hbar\omega}{2}}$$

$\hat{a}|0\rangle = 0$

ground state.

$$\hat{H}\hat{a}^{\dagger} = \hat{a}^{\dagger}\hat{H} \underbrace{- \hat{a}^{\dagger}\hat{H} + \hat{H}\hat{a}^{\dagger}}_{[\hat{H}, \hat{a}^{\dagger}]}$$

energy increase $\hbar\omega$

$$\hat{H}\hat{a}^{\dagger}|E_n\rangle = \hat{a}^{\dagger}\hat{H}|E_n\rangle + [\hat{H}, \hat{a}^{\dagger}]|E_n\rangle = \hat{a}^{\dagger}E_n|E_n\rangle + \hbar\omega\hat{a}^{\dagger}|E_n\rangle = (E_n + \hbar\omega)(\hat{a}^{\dagger}|E_n\rangle)$$

$$\hat{H}\left(\hat{a}^{\dagger}(\hat{a}^{\dagger}|E_n\rangle)\right) = \underbrace{(E_n + \hbar\omega + \hbar\omega)}_{E_n + 2\hbar\omega}\left[\hat{a}^{\dagger}(\hat{a}^{\dagger}|E_n\rangle)\right]$$

starting with ground state: $\frac{\hbar\omega}{2}, \frac{3\hbar\omega}{2}, \frac{5\hbar\omega}{2}$

$$\hat{H}|0\rangle = \frac{\hbar\omega}{2}|0\rangle \Rightarrow \hat{H}\hat{a}^{\dagger}|0\rangle = \left(\frac{\hbar\omega}{2} + \hbar\omega\right)|0\rangle$$

$$\hat{H}\hat{a}^{\dagger}\hat{a}^{\dagger}|0\rangle = \left(\frac{\hbar\omega}{2} + 2\hbar\omega\right)|0\rangle$$

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega}$$

Q9. a) cont $\boxed{\text{for 3D}}$ $H = \hbar\omega \sum_{i=1}^{3} \left(a_i^\dagger a_i + \frac{1}{2}\right)$

$$E_0 = \langle 0|\hat{H}|0\rangle = \hbar\omega\left[\langle 0|\sum_{i=1}^{3}\left(a_i^\dagger a_i + \frac{1}{2}\right)|0\rangle\right] = \hbar\omega\left[\langle 0|\cancel{\sum_{i=1}^{3} a_i^\dagger a_i}|0\rangle + \langle 0|\frac{3}{2}|0\rangle\right]$$

$$\boxed{E_0 = \frac{3\hbar\omega}{2}}$$

$$+\hat{H}\sum_{i=1}^{3} a_i^\dagger - \sum_{i=1}^{3} a_i^\dagger \hat{H} = \left[\hat{H}, (a_1^\dagger + a_2^\dagger + a_3^\dagger)\right] = \hbar\omega \sum_{i=1}^{3} \hat{a}_i^\dagger$$

$$\therefore \Rightarrow \boxed{E_n = E_x + E_y + E_z = \left(n_x + n_y + n_z + \frac{3}{2}\right)\hbar\omega}$$

b) degenerescência: $n = n_1 + n_2 + n_3$; $n_2 + n_3 = n - n_1$

n	E	degenerescência
0	$\frac{3\hbar\omega}{2}$	0
1	$\frac{5}{2}\hbar\omega$	3
2	$\frac{7}{2}\hbar\omega$	6

1, 0, 0
0, 1, 0
0, 0, 1

2, 0, 0
0, 2, 0
0, 0, 2
1, 1, 0
1, 0, 1
0, 1, 1

$\frac{1}{2}(n+1)(n+2)$

c) $$\nabla^2\psi = \left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) - \frac{L^2}{\hbar^2 r^2}\psi\right] \qquad L^2 Y_{lm} = \hbar^2 l(l+1) Y_{lm}$$

$$-\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) - \frac{L^2}{\hbar^2 r^2}\psi\right] + \frac{1}{2}m\omega^2 r^2 = E\psi$$

$$-\frac{\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)+\frac{\hat{L}^2}{2mr^2}\Psi+\frac{1}{2}m\omega^2r^2\Psi = E\Psi \qquad u = rR \qquad du = R + \frac{\partial R}{\partial r} \qquad du = \frac{\partial R}{\partial r}+\frac{\partial^2 r}{\partial r^2}$$

Let $\boxed{\Psi_{nlm} = R_{nl}(r)\cdot Y_{lm}(\theta,\phi)}$ $\qquad \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} = \frac{1}{r^2}\left(2r\frac{\partial}{\partial r}+r^2\frac{\partial^2 R}{\partial r^2}\right)$

$$Y_{lm}(\theta,\phi)-\frac{\hbar^2}{2mr^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R_{nl}(r)}{\partial r}\right)+\Bigg[\underbrace{\frac{1}{2}m\omega^2r^2+\frac{\hbar^2 l(l+1)}{2mr^2}}_{V_{eff}}\Bigg]Y_{lm}\cdot R(r) = EYR$$

$$= -\frac{\hbar^2}{2mr^2}\left[2r\cdot\frac{dR(r)}{dr}+r^2\frac{d^2R(r)}{dr^2}\right]+\left(\frac{1}{2}m\omega^2r^2+\frac{\hbar^2 l(l+1)}{2mr^2}\right)R(r) = E\,R(r)$$

d) $l=0$, $R(r) = e^{-\alpha r^2}$; $\frac{dR}{dr} = -2r\alpha e^{-\alpha r^2}$; $\frac{d^2R}{dr^2} = -2\alpha e + 4r^2\alpha^2 e^{-\alpha r^2}$

$$-\frac{\hbar^2}{m}\left[-2r\alpha e^{-\alpha r^2}+\left(-2r^2\alpha e^{-\alpha r^2}+4r^4\alpha^2 e^{-\alpha r^2}\right)\right]+\left[\frac{1}{2}m\omega^2r^2\right]\cdot \cancel{e^{-\alpha r^2}} = E_n \cancel{e^{-\alpha r^2}}$$

$$E_n = \frac{2\hbar^2 r\alpha}{m}+\frac{2\hbar^2 r^2\alpha}{m}-\frac{4\hbar^2 r^4\alpha^2}{m}+\frac{1}{2}m\omega^2r^2$$

if $-\frac{\hbar^2}{2m}\frac{d^2 u}{dr^2}+ uV u(r) = E\,u(r)$ $\qquad u(r) = r\cdot e^{-\alpha r^2}$

$\boxed{\text{process other } \boxed{E}}$? α so α substitutes E ?

Q10. sistem continuo $F = U - TS = -N_0 kT \ln Z$

$$Z = \int_V \cdots \iint_p \cdots \int e^{-\beta \sum_{i=1}^{N} \frac{\vec{p}_i^2}{2m}} \, d\vec{p}^{\,3}_{1\ldots N} \, d\vec{r}^{\,3}_{1\ldots N}$$

$$= \int_V \cdots \iint_p \cdots \int e^{-\beta\frac{\vec{p}_1^2}{2m}} \cdot e^{-\beta\frac{\vec{p}_2^2}{2m}} \cdots e^{-\beta\frac{\vec{p}_N^2}{2m}} \, d\vec{p}_1 \ldots d\vec{p}_N^{\,3} \, d\vec{r}^{\,3}_{1\ldots N}$$

Para cada partícula

$$\int_{-\infty}^{\infty} e^{-\frac{\beta p^2}{2m}} dp = \sqrt{\frac{\pi \, 2m}{\beta}} \qquad \lambda = (2\pi m kT)^{\frac{1}{2}}$$

para $p_x, p_y, p_z \Rightarrow Z = \left(\frac{2m\pi}{\beta}\right)^{\frac{3}{2}} \cdot \int \cdots \int d\vec{r}^{\,3}_{1,N} \Rightarrow Z = \frac{V}{\lambda^3}$

b) sistem N átomos

$$\boxed{Z = \frac{V^N}{\lambda^{3N}}} \Rightarrow Z = \frac{1}{N! \, h^{3N}} \left(\frac{V}{\lambda^3}\right)^N$$

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a) $H = (p_x^2 + p_y^2 + p_z^2)/2m$

$$Z = \int_V \iint e^{-\beta \sum_{i=1}^{3} p_i^2/2m} \, dp_i^3 \, dr_i^3 = \int \iint \cdots \int e^{-\beta\frac{p_x^2}{2m}} \cdot e^{-\frac{\beta p_y^2}{2m}} \cdot e^{-\frac{\beta p_z^2}{2m}} \, dp_x^3 \, dp_y^3 \, dp_z^3 \, dr_i^3$$

$$\int_{-\infty}^{\infty} e^{-\frac{\beta p^2}{2m}} dp = \sqrt{\frac{\pi \cdot 2m}{\beta}} \Rightarrow Z = \frac{1}{h^3}\left(\frac{2m\pi}{\beta}\right)^{\frac{3}{2}} \cdot V$$

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b) $Z = \frac{1}{N!}\left(\frac{2\pi m k_B T}{h^2}\right)^{3N/2} \cdot V^{3N}.$

$$F = -\frac{1}{\beta}\ln Z = -\frac{1}{\beta}\cdot\left[\frac{3N}{2}\ln\left(\frac{2\pi m k_B T}{h^2 \beta}\right) + 3N\ln V - \ln N!\right]$$

$$\Rightarrow f = \frac{F}{N} = \frac{-3k_B T}{2}\ln\left(\frac{2\pi m k_B T}{h^2}\right) - \cancel{3} k_B T \ln V + \frac{k_B T}{N}\ln(N!)$$

0 l'hôpital $N \to \infty$

c) $$U = -\frac{\partial}{\partial\beta}\ln Z = \boxed{\frac{3}{2}k_B T}$$

$$\frac{3}{2}\cdot\frac{\cancel{h^2\beta}}{\cancel{2\pi m}}\cdot\left(-\frac{2\pi m}{h^2\beta^2}\right) = \frac{3}{2}k_B T$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = +\frac{3N}{\beta V} \Rightarrow \boxed{\frac{\cancel{3} k_B T}{v}}$$

d) $$S = -\left(\frac{\partial F}{\partial T}\right)_V = k_B\frac{3}{2}\ln T + k_B\ln v + \frac{3}{2}k_B + k_B C.$$